3.4: THE REPRESENTATION OF WAVEGUIDES CONTAINING SMALL FERRIMAGNETIC ELLIPSOIDS*

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As ferrimagnetic materials having lower and lower intrinsic losses have become available, it has become feasible to build microwave devices in which small ellipsoidal samples are placed in uniform propagating structures. In many such applications, the sample, although small physically, presents a large electrical discontinuity, so that the usual perturbation procedures [1,2] are no longer adequate to describe the interaction between the sample and the microwave circuit, and a more complete analysis must be made.

We have carried out an approximate analysis of the case of a uniform waveguide of arbitrary cross-section containing an ellipsoidal sample whose dimensions are small compared with a free-space wavelength. The dc field is assumed to be along one of the principal axes of the ellipsoid, and to be perpendicular to the direction of propagation in the waveguide. The method used is an approximate one, similar to that used by Turner [3] for the special case of a spherical sample, in rectangular waveguide, in which only the dominant modes of the sample and waveguide are considered. The effect of the neglected modes is to produce a reactive loading on the sample which slightly alters its resonant frequency.

If the waveguide propagates in the y-direction, and contains a sample saturated by a dc field along the z-axis, Figure 1, the scattering matrix, referred to the cross-sectional plane containing the sample, has the form

$$\overleftrightarrow{S} = F \begin{pmatrix} \frac{j\beta V_s}{2}\left(\chi_{11}h_x^2 + \chi_{22}h_y^2\right) & 1 - j\beta V_s\chi_{12}h_xh_y \\ 1 + j\beta V_s\chi_{12}h_xh_y & \frac{j\beta V_s}{2}\left(\chi_{11}h_x^2 + \chi_{22}h_y^2\right) \end{pmatrix}, \qquad (1)$$

where $F = \dfrac{1}{1 + \frac{j\beta V_s}{2}\left(\chi_{11}h_x^2 - \chi_{22}h_y^2\right)}$

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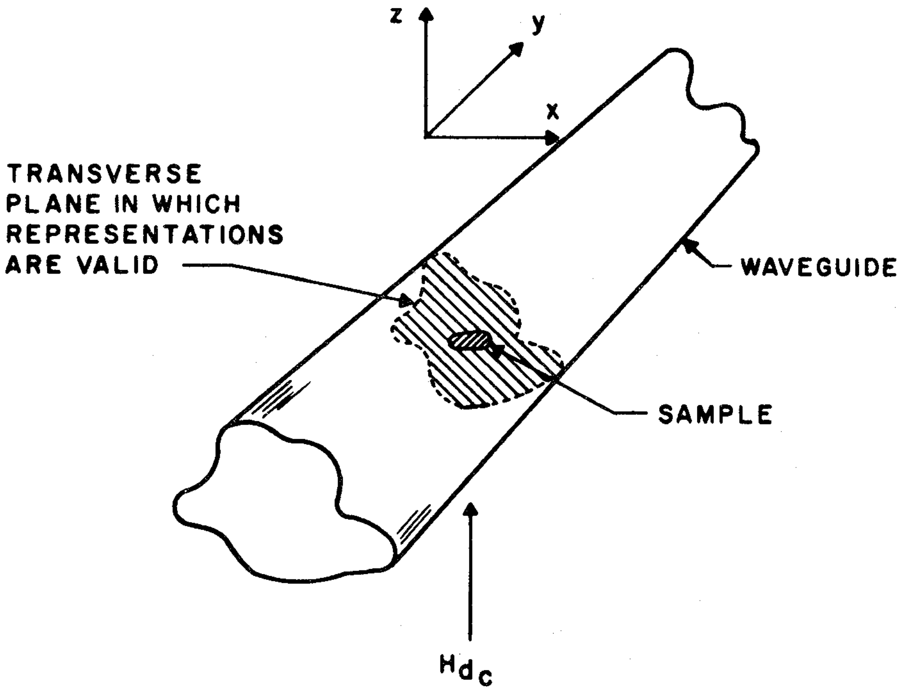


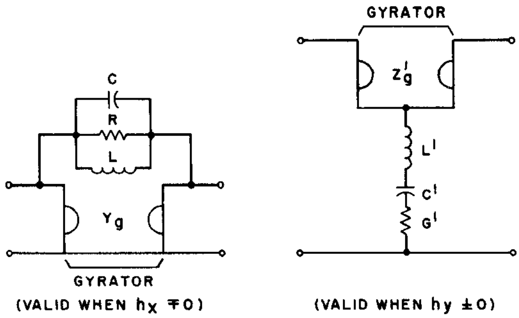
Fig. 1. Coordinate system used to discuss scattering from ferrimagnetic ellipsoids in waveguides.

Here β is the propagation constant of the waveguide, V_s the volume of the sample, and the external rf susceptibility (ratio of the magnetization to the external rf field) is assumed to have a general form,

$$\overleftrightarrow{\chi}_{ext} = \begin{pmatrix} \chi_{11} & \chi_{12} & 0 \\ -\chi_{12} & \chi_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The quantities h_x and h_y are components of the magnetic field eigenvector of the propagating mode, which is proportional to the actual waveguide field, but normalized so that the square of its transverse component integrates to unity over the waveguide cross-section.

Lumped-element equivalent circuit representations may be obtained from the specific form of the external susceptibility tensor. Two alternative circuits, with element values, are given in Figure 2. In these expressions, ω_o is the ferrimagnetic resonance frequency of the sample, Q_f its unloaded Q, which is determined by the observed linewidth, ΔH;



ELEMENT VALUES

$$\frac{1}{LC} = \frac{1}{L'C'} = \omega_0^2 = (\omega_H + N_x\omega_M)(\omega_H + N_y\omega_M)$$

$$Q_f = \omega_0 RC = \frac{\omega_0 L'}{R'} = \frac{\omega_0^2}{\mu_0\gamma\Delta H} \quad \frac{1}{\omega_H + \frac{\omega_M}{2}(N_x + N_y)}$$

$$\frac{R}{Z_0 Q_f} = \frac{\beta h_x^2 v_s \omega_M}{\omega_H + N_x\omega_M} \qquad \frac{G'}{Y_0 Q_f} = \frac{\beta |h_y|^2 v_s \omega_M}{\omega_H + N_y\omega_M}$$

$$\frac{Y_g}{Y_0} = \frac{-\omega}{\omega_H + N_y\omega_M}\,\frac{|h_y|}{h_x} \qquad \frac{Z_g'}{Z_0} = \frac{\omega}{\omega_H + N_x\omega_M}\,\frac{h_x}{|h_y|}$$

Fig. 2. Lumped element networks for ferrimagnetic ellipsoids in waveguides.

μ_o is the permeability of free space (MKS units are used), γ is the gyromagnetic ratio, N_x and N_y are the transverse demagnetizing factors, and ω_H and ω_M are $\mu_o\gamma$ times the internal dc field and saturation magnetization, M_s, respectively. The impedances are normalized to the wave impedance, Z_o, of the waveguide.

Several important aspects of the preceding theory have been verified by experiments conducted at X-band on spheres and discs of single crystal yttrium iron garnet in rectangular waveguide.

The results of two such experiments carried out on spherical samples placed in shorted sections of waveguide and biased to ferrimagnetic resonance, are shown in Figures 3 and 4. In Figure 3 is shown the result of a measurement, at a number of frequencies, of the quantity $\frac{R}{Z_o}(\lambda_g \Delta H)$, where $\frac{R}{Z_o}$ is the measured normalized input impedance of the waveguide, and λ_g is the waveguide wavelength. This quantity should, according to the theory, have the constant value shown. The sample used was approximately critically coupled to the waveguide, and so presented a large electrical discontinuity which, nonetheless, is adequately described by the present simple theory. Figure 4 shows the result of a measurement of the power absorbed by a YIG sphere as a function of the distance from the sample to the transverse shorting plane. Agreement with the theoretical curve, obtained from the known sample parameters

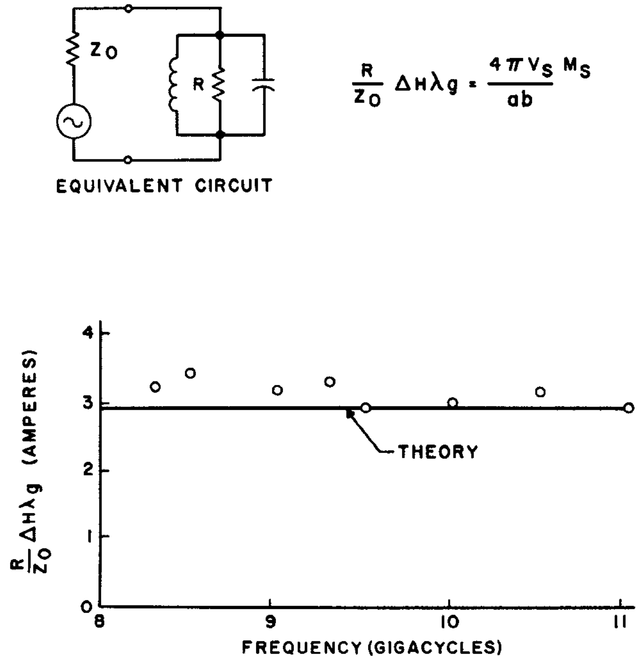


Fig. 3. Coupling of a YIG sphere to rectangular waveguide, as a function of frequency.

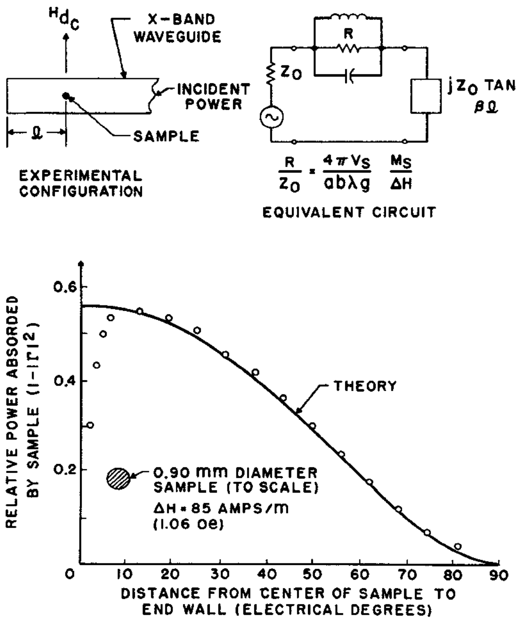


Fig. 4. Power absorbed by a YIG sphere in a shorted waveguide, as a function of its longitudinal position.

with the aid of the equivalent circuit shown, is excellent, except where the sample is within one or two diameters of the shorting plane [4]. In this region it is necessary to consider the interaction between the near fields of the sample and the waveguide wall. If this is done correctly, taking into account the finite conductivity of the wall, and if allowance is made for the indirect excitation of other magnetostatic modes in the sample, experiment and theory can be brought into agreement for a sample placed arbitrarily close to a conducting wall.

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1. P. S. Epstein and A. D. Berk, "Ferrite Post in a Rectangular Waveguide," J. Appl. Phys. 27, 1328 (1956).

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3. E. H. Turner, Bull. Am. Phys. Soc. 6, 159 (1961), and unpublished work.

4. E. G. Spencer and R. C. LeCraw, "Wall Effects on Microwave Measurements of Ferrite Spheres," J. Appl. Phys. 26, 250 (1955).